## 2. Geometry

Olsen and Thomas have observed that their water jet leaves a clean cut in sandstone, more suggestive of erosion than of gross internal failure. One can therefore make some expedient assumptions about the geometry of the cut without straying far from reality. The idealized cut is shown in Fig. 2. The jet is presumed to enter with a square cross-section of width and depth $d_o$, and to leave behind a uniform slot of width $d_o$. Cutting thus takes place entirely on the forward face of the jet, where the surface pressure due to streamline curvature is greatest. Friction against the cutting surface decelerates the jet, and the stream must deepen to accommodate the constant volume flow of water. The local depth d increases station-by-station downstream.

Hopefully the reader will not be put off by the geometrical assumptions, which may appear sweeping. The assumption of a square jet may seem strange, especially since $d_o$ is identified later with nozzle diameter, and the cut is never quite so narrow as the nozzle diameter itself. The purpose of this paper is to frame the physics of hydraulic rock cutting in a geometry that admits simple analysis. The solution for h will contain the empirical universal constant $\mu_w$, which can absorb minor geometrical deficiencies.

Figure 3 illustrates the geometrical properties of the cutting surface. The surface follows the curve y(x), where the ordinate y increases downward into the stone, and the abscissa x increases backward along the cut. The origin of the coordinate system is the point where the jet first impacts the stone. The arc-length s specifies location along the cutting surface, and $\theta$ is the local angle of the cut with respect to the horizontal. Note that the jet can enter at an angle $\theta_o$ different than 90° and is so illustrated. Experiments have been carried out to date under conditions of normal impingement, $\theta_o = 90°$, but the theory indicates that impingement angles $\theta_o$ greater than 90° will produce deeper cuts (cf. Section 8).

The quantity connecting geometry to dynamics is the local radius of curvature of the cutting surface,

$$R = -\frac{ds}{d\theta} . \tag{2}$$

Now x and y can be expressed in terms of R as follows. Note that

$$\frac{dy}{ds} = \frac{dy}{d\theta}\frac{d\theta}{ds} = \sin\theta ,$$